Problem 3 (25%)

An XOR-OR Boolean formula is constructed from the following rules:

- the Boolean constants 0 (false) and 1 (true),
- Boolean variables: x_1, x_2, ..., x_n,
- the Boolean operators: $\oplus$ (exclusive-or) and $\vee$ (inclusive-or), and
- parenthesis.

The Boolean operator $\oplus$ is the exclusive-or operator, given by the following truth table:

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

For example, the formula

$$\phi = (x_1 \oplus x_2) \oplus (1 \vee x_1)$$

is an XOR-OR formula. The decision problem associated with the satisfiability of XOR-OR Boolean formulas is

$$\text{XOR-OR-SAT} = \{\langle\phi\rangle : \phi \text{ is a satisfiable XOR-OR Boolean formula }\}.$$

For example,

$$\phi = (x_1 \oplus x_2) \oplus (1 \vee x_1)$$

belongs to XOR-OR-SAT since ϕ is satisfiable (for example, for the truth assignment $x_1 = 1, x_2 = 1$).

Question 3.1

Show that XOR-OR-SAT $\in$ NP.

Question 3.2

Show that XOR-OR-SAT is NP-complete. Hint: you may have to use the identity: $P \wedge Q \Leftrightarrow \neg(\neg P \vee \neg Q)$.

Question 3.3

Professor C. Lever claims that he has an algorithm which solves the decision problem for XOR-OR Boolean formulas in time $O(n^3 \log\log 2^{2^n})$ where n is the length of the formula. Discuss whether this claim sounds reasonable.

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