IT-math F2003 : Classroom Exercises
Episode 10, April 8, 2003

1. Let $f(n) = n^2 + 3n + 4$ and $g(n) = n^2$. Show that $g(n) = O(f(n))$.
2. Suppose $c \in \mathbb{R}$ and $c \neq 0$. Show that $c \cdot f = \Theta(f)$.
3. Show that the binary relation defined by big-O is transitive: If $f = O(g)$ and $g = O(h)$ then $f = O(h)$.
4. Show that the binary relation $f = \Theta(g)$ on $\mathbb{R}^{\mathbb{N}}$ is an equivalence relation.
5. Show that $\sqrt{n} = o(n)$.
6. Show that
   (a) If $f = o(g)$ and $h = o(g)$ then $f + h = o(g)$;
   (b) If $f = O(g)$ and $h = O(g)$ then $f + h = O(g)$.

IT-math F2003 : Homework Exercises
Episode 10, April 8, 2003

Fisherperson's Exercises

1. Let us look at four-letter words that only use letters A, B, C, D, E, F.
   (a) How many words use the letter A exactly twice?
   (b) How many words have their letters in alphabetical order and without repetitions?
   (c) How many words use two C's and two E's?
   Motivate your answers.

2. Show that:
   (a) $n = o(n \cdot \log n)$;
   (b) $2^n = O(3^n)$.

3. Prove that if $f = O(g)$ and $g = o(h)$, then $f = o(h)$.

Snake-Charmer's Exercises

1. Prove or refute:
   (a) $n \cdot \log n = o(n^2)$;
   (b) $2 \log n + 4n + 3n \log n = o(n \cdot \log 2n)$;
   (c) $3^n = O(2^n)$;
   (d) $n! = o(n^n)$;
   (e) $n = \Theta(\lceil n \rceil)$.

2. Let $\alpha > \beta > 0$ be real numbers. Show that $n^\beta = o(n^\alpha)$.

3. (for those familiar with Java or a similar programming language) Consider the following fragment of Java code:

```
int x = 0;
for (i = 1; i<=n; i++) {
    for (j = 1; j<=i; j++) {
        x++;
    }
}
```

   If n is the value of an `int` varaible `n` immediately before the execution of this fragment, $n > 0$, and $x(n)$ is the value of `x` immediately after the execution (considered as a function of n), show that $x(n) = \Theta(n^2)$.

   [In this exercise, assume that Java does perfect integer arithmetic rather than arithmetic mod 2^{32}.]

Lion-Hunter's Exercises

1. Give an example of three functions $f, g, h : \mathbb{N} \to \mathbb{R}$ such that $f = \Omega(g)$, $h = \Omega(g)$, yet $f + h \neq \Omega(g)$. (Compare with Classroom Exercise 6.)

2. Prove or refute: If $f(n) = \Theta(g(n))$ then $2^{f(n)} = \Theta(2^{g(n)})$.

3. Assume the functions f and g take on positive values only. Show that $f(n)+g(n) = \Theta(h(n))$, where the function h is defined by $h(n) = \max\{f(n), g(n)\}$.

Dragonslayer's Exercise

1. Prove or refute: For any functions $f, g : \mathbb{N} \to \mathbb{R}$, $f(n) = O(g(n))$ implies $f(n+1) = O(g(n))$.