IT-math F2003 : Classroom Exercises
Episode 3, February 18, 2003

1. Let $A = \{1, 2\}$, $B = \{3, 4, 5\}$, $C = \{2, 3, 4, 5\}$. Calculate (i.e. list explicitly the elements of) $A \cup B$, $A \cup C$, $A \cap B$, $A \cap C$, $A \setminus B$, $A \setminus C$, $A \triangle B$, $A \triangle C$.
2. Let $A = \{1, 2, 3, 6\}$, $B = \{3, 4, 5\}$, $C = \{1, 3, 5, 7\}$. Draw the Venn diagram for the sets A, B, C, and place the numbers $1, \ldots, 9$ in the appropriate regions of the diagram.
3. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ hold for any sets A, B, C.
4. Show that $A \triangle (B \triangle C) = (A \triangle B) \triangle C$ for any sets A, B, C.
5. Calculate $A \cap \emptyset$, $A \cup \emptyset$, $A \triangle \emptyset$, $A \setminus \emptyset$, $\emptyset \setminus A$.
6. Let $A \subseteq X$, and let A^c denote the complement of A relative to X. Show that $A \cap A^c = \emptyset$, $A \cup A^c = X$, $A \triangle A^c = X$, $(A^c)^c = A$.
7. Let $A, B \subseteq X$, and, for any subset Y of X, let Y^c denote the complement of Y relative to X. Prove that $(A \cap B)^c = A^c \cup B^c$.
8. Suppose $|A| = n$ (i.e. the set A has exactly n elements). Show that $|\mathcal{P}(A)| = 2^n$. (Recall that $\mathcal{P}(A)$ is the power set of A).
9. Let $A = \{1, 2\}$, $B = \{1, 3, 5\}$. Calculate $A \times B$, $B \times A$, $A \times A \times A$.
10. Suppose that $|A| = n$ and $|B| = m$. Show that $|A \times B| = n \cdot m$.
11. Give an example of sets A, B such that $A \times B \neq B \times A$.
12. Give an example of sets A, B, C such that the equality $A \cup (B \times C) = (A \cup B) \times (A \cup B)$ does *not* hold. Also give an example of sets A, B, C such that this equality *does* hold.

IT-math F2003 : Homework Exercises
Episode 3, February 18, 2003

to be submitted on or before February 25, 2003

Fisherperson's Exercises

1. Let $A = \{1, 3, 6\}$, $B = \{2, 3, 7\}$, $C = \{1, 2, 4\}$, $X = \{1, \ldots, 7\}$, the complement $(\cdot)^c$ below being relative to X. List the elements of
   (a) $A \setminus (B^c \setminus C)$;
   (b) $A \cap (C^c \cup B)^c$;
   (c) $(A \cap C) \setminus (B \cap C)$;
   (d) $(X \setminus A) \vartriangle (B^c \setminus C)$.

2. Find out whether the following are true (and motivate your answer):
   (a) $\{1, 36\} = \{1, \binom{7}{4}\}$;
   (b) $\{2, 3\} \subseteq \{\{2, 2, 3\}\}$;
   (c) $|\{x \in \mathbb{R} \mid x^2 + x = 2\}| = |\{2, 2, 3\}|$;
   (d) $\{(1, 2), (2, 1)\} = \{(2, 1), (1, 2)\}$.

3. Show that $A \cap (A \cup B) = A$ and $A \cup (A \cap B) = A$ for any sets A and B.

Snake-Charmer's Exercises

1. Prove or refute: For any sets A, B, C we have $(A \setminus B) \setminus C = A \setminus (B \setminus C)$.

2. Prove that, for any sets A, B with $A, B \subseteq X$ and the complement $(\cdot)^c$ being relative to X, the following statements are equivalent (that is, any two of the following statements can be connected by an 'iff'):
   (i) $A \subseteq B$;
   (ii) $A \cap B = A$;
   (iii) $A \cup B = B$;
   (iv) $B^c \subseteq A^c$.

3. Show that for any sets A, B, C one has $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

Lion-Hunter's Exercises

1. Write a careful proof (no Venn diagrams) that $(A \cup B)^c = A^c \cap B^c$ for any sets A, B, where the complement is taken relative to some set X such that $A, B \subseteq X$.

2. For finite sets A and B, show $|A \cup B| = |A| + |B| - |A \cap B|$.

3. For all sets A, B, C, prove that if $A \times B = A \times C$, then $A = \varnothing$ or $B = C$.

Dragonslayer's Exercise

1. Refute: There is a set X such that $X = \mathcal{P}(X)$.