IT-math F2003 : Classroom Exercises
Episode 4, February 25, 2003

1. Is the number 417 prime? Is the number 419 prime?
2. Show that for positive integers n one has $\gcd(n, 0) = n$.
3. Show that for any $k \in \mathbb{Z}$, if $n \mid a$ and $n \mid b$, then $n \mid a - k \cdot b$.
4. Find $\gcd(437, 621)$ using Euclid's Algorithm.
5. Find integers a and b such that $437a + 621b = 23$.
6. Let $n = p_1^{\alpha_1} \cdot \cdots \cdot p_n^{\alpha_n}$, and $m = p_1^{\beta_1} \cdot \cdots \cdot p_n^{\beta_n}$ be prime factorizations of m and n respectively, where we assume that $p_1, \cdots, p_n$ are the first n primes (in, say, increasing order), and α_i, β_i are non-negative integers. Show that

$$\gcd(m, n) = p_1^{\min(\alpha_1,\beta_1)} \cdot \cdots \cdot p_n^{\min(\alpha_n,\beta_n)}.$$

IT-math F2003 : Homework Exercises
Episode 4, February 25, 2003

to be submitted on or before March 4, 2003

Fisherperson's Exercises

1. Find a prime factorization of 3901.
2. (a) Find $\gcd(359, 2349)$;
   (b) Find integers x and y such that $359x + 2349y = 1$.
3. Show that if $a, b, x, y \in \mathbb{Z}$ and $(a, b) \neq (0, 0)$, then $\gcd(a, b) \mid ax + by$.

Snake-Charmer's Exercises

1. Let $a, b, k \in \mathbb{Z}_+$. Show that $\gcd(k \cdot a, k \cdot b) = k \cdot \gcd(a, b)$.
2. Recall the Fibonacci numbers defined by the recursions $f_0 = 0$, $f_1 = 1$ and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 0$. Show that $\gcd(f_n, f_{n+1}) = 1$ for all $n \in \mathbb{N}$.
3. Show that if $a, b > 1$, $\gcd(a, b) = 1$, $a \mid n$, and $b \mid n$, then $a \cdot b \mid n$.

Lion-Hunter's Exercises

1. If the integers x and y are solutions to $a \cdot x + b \cdot y = \gcd(a, b)$, show that x and y are relatively prime (i.e. $\gcd(x, y) = 1$).
2. Show that if $2^n - 1$ is prime then so is n.
   (Hint: Use the formula $x^m - 1 = (x - 1)(x^{m-1} + x^{m-2} + \cdots + x + 1)$.)
3. Show that if a is an odd integer then $a^2 - 1$ is divisible by 8.

Dragonslayer's Exercise

1. Let $a, b, c, d \in \mathbb{Z}_+$. Show that the equation $ax + by + cz = d$ admits integer solutions x, y, z if and only if $\gcd(\gcd(a, b), c) \mid d$.