IT-math F2003 : Classroom Exercises
Episode 6, March 11, 2003

1. (a) What is $(1011)_2$ in decimal?
   (b) Represent 27 in binary;
   (c) Rewrite $(3456)_7$ in decimal;
   (d) Find the base 9 representation of 823.

2. Find the sum of $(1001101)_2$ and $(1110110001)_2$ in binary without converting to the decimal notation.

3. Let A and B be finite sets with $|A| = n$ and $|B| = m$. How many relations between A and B are there?

4. (a) Draw the digraph of the relation R on $\mathbb{Z}_{16}$, where R is defined by

$$a \; R \; b \quad \Longleftrightarrow \quad a \text{ is the inverse of } b \text{ in } \mathbb{Z}_{16}.$$

   (b) Is the relation R from clause (a) reflexive? irreflexive[1]? symmetric? antisymmetric? transitive?

5. **(C. Pedersen)** Consider the relation Q on $\mathbb{R}^2$ defined by

$$(x, y) \; Q \; (z, w) \quad \Longleftrightarrow \quad x \leq z \text{ and } y \leq w.$$

   Show that Q is
   (a) reflexive;
   (b) antisymmetric.

IT-math F2003 : Homework Exercises
Episode 6, March 11, 2003

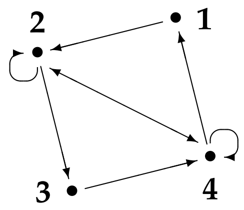
Fisherperson's Exercises

1. Does the expression 1001100010 represent a number in binary notation? in octal (i.e. base-8) notation? in hexadecimal notation?

2. Find the sum, in hexadecimal notation, of the numbers $(49F7)_{16}$ and $(C66)_{16}$.

[1] A binary relation S on a set A is *irreflexive* if there are no $a \in A$ with $a \; S \; a$.

3. Consider the relation R on $\{1,2,3,4\}$ whose digraph is given by the following picture:



Write down the relation R as a set of pairs. Is R transitive?

4. Draw the digraph of the relation S on $\{1,2,3,4,5\}$, where

$$S = \{\, (m,n) \in \{1,2,3,4,5\}^2 \mid m + n \equiv 0 \pmod 3 \,\}.$$

Snake-Charmer's Exercises

1. Let $b > 1$ be an integer, $K \in \mathbb{N}$, and let n_i be some base-b digits: $0 \le n_i < b$. Show that $\sum_{i=0}^{K} n_i \cdot b^i < b^{K+1}$.

   [Suggestion: Induction on K.]

2. Give an example of a relation on $\mathbb{N}$ that is both symmetric and antisymmetric.

3. **(C. Pedersen)** Recall the relation Q on $\mathbb{R}^2$ defined by

$$(x,y)\ Q\ (z,w) \quad \Longleftrightarrow \quad x \le z \text{ and } y \le w.$$

   Show that Q is transitive.

Lion-Hunter's Exercises

1. Let $(n_K n_{K-1} \ldots n_1 n_0)_{10}$ be the decimal representation of the number $n = \sum_{i=0}^{K} n_i \cdot 10^i$. Show that $3 \mid n$ if and only if $3 \mid \sum_{i=0}^{K} n_i$.

   [Hint: Compare n and $\sum_{i=0}^{K} n_i$ modulo 3.]

2. Suppose R_1 and R_2 are partial orderings on a set X. Show that $R_1 \cap R_2$ is a partial ordering on X.

3. (for those familiar with Java or a similar programming language) How many lines of output will the following fragment of Java code produce?

```
int i = 1;
while (i != 10) {
    System.out.println("it wont take a moment");
    i = i + 7;
}
```

Dragonslayer's Exercise

1. Find three sets X_1, X_2, X_3 with the following properties:
   (i) If $x \in X_i$ for some i with $1 \le i \le 3$ then $x \in \{X_1, X_2, X_3\}$, and
   (ii) The relation $\in$ is a transitive relation on $\{X_1, X_2, X_3\}$.