

# IT-math F2003 : Exercises

## Episode 1, February 4, 2003

### Homework Exercises

to be submitted on or before February 11, 2003

#### Fisherperson's Exercises

1. Calculate (a)  $3!$ ;  
(b)  $7!$ .
2. Calculate (a)  $\binom{0}{0}$ ;  
(b)  $\binom{5}{3}$ ;  
(c)  $\binom{7}{4}$ .

#### Snake-Charmer's Exercises

1. Try to find and write down a careful proof of the following statement:  
"Let  $a$  and  $b$  be any integers. If  $a \geq b \geq 0$  and  $c \geq d \geq 0$ , then  $a \cdot c \geq b \cdot d$ ."
2. Prove by induction on  $n$  (starting with  $n = 2$ ) that  $\binom{n}{2} = \frac{n \cdot (n-1)}{2}$  for all integers  $n \geq 2$ .

#### Lion-Hunter's Exercises

1. Prove by induction on  $n \geq 0$  that  $\binom{n}{k} = \binom{n}{n-k}$  for all integers  $k$  such that  $0 \leq k \leq n$ .
2. Draw a generous number of rows of the Pascal triangle, and underline (or highlight, or encircle etc.) all even entries. Notice that there is a pattern. In which rows does one find odd entries only? Try to formulate a plausible conjecture of the form  
"All entries in the  $n$ th row of the Pascal triangle are odd if and only if  $n \dots$ "

#### Dragonslayer's Exercise

1. Show by induction on  $n \geq 1$  that  $3^n + 7^n - 2$  is divisible by 8.