

Recursion Theory F2003 : Homework Exercises

Episode 7, March 20, 2003

1. Call a set A *self-dual* if $A \leq_m \bar{A}$ (\bar{A} is the complement of A). Show that no index set can be self-dual.
2. Show that any individual partial recursive function, as well as any individual r.e. set have infinitely many indices.
3. Show that $\{e \mid \{e\}(e) \downarrow\}$ is not an index set.
4. Let $\varphi(x)$ be a partial recursive function. Show that there exists a total recursive function $f(x)$ such that $W_{\varphi(x)} = W_{f(x)}$ for all $x \in \text{dom } \varphi$.
5. Show that the set $\{x \mid \exists y (y \in W_x \ \& \ |W_y| = \infty)\}$ is Σ_3^0 -complete.
6. Show that any Σ_2^0 set is reducible to FIN , $\overline{\text{TOT}}$, where $\text{FIN} = \{e \mid |W_e| < \infty\}$ and $\overline{\text{TOT}} = \{e \mid |W_e| \neq \mathbb{N}\}$.