

Recursion Theory F2003 : Exercises

Episode 1, February 6, 2003

Homework Exercises

1. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be total and computable and let $X \subseteq \mathbb{N}$ be computable. Show that $f^{-1}[X]$ is also computable.
2. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be total computable and injective. Show that the (partial) function f^{-1} is computable.
3. Let P be a basic machine (thus, no macros) program that computes a total function f , and does not contain instructions of the form DECREASE. . . . Show that f is a constant function.
4. Show that the binary relation $x = y$ is recursive.
5. Suppose $f, g : \mathbb{N} \rightarrow \mathbb{N}$ are both total computable, g injective, $f(n) \geq g(n)$ for all $n \in \mathbb{N}$, and that $\text{rng } g$ computable. Show that $\text{rng } f$ is then also computable.
6. Let $e = 2.a_1a_2a_3 \dots$ be the decimal expansion of e . Show that the function $i \mapsto a_i$ is computable.